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Fatigue Limits of Steels and Stress Gradient*

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Пределы выносливости сталей и градиент напряжений

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На основании обобщения результатов многочисленных экспериментальных исследований предложены и обоснованы эмпирические соотношения между пределами выносливости сталей и градиентом напряжений, в которых учитывается накопление усталостного повреждения в гладких образцах и образцах с концентраторами напряжений. Предложен метод расчета параметров этих соотношений с учетом механических свойств сталей. Получено хорошее соответствие между предложенными соотношениями и экспериментальными результатами.

Ключевые слова: многоцикловая усталость, предел выносливости, градиент напряжений.

Introduction. Numerous experimental investigations on high-cycle fatigue of metals and alloys show a significant increase in the fatigue limit with an increase in the stress gradient that characterizes the nonuniformity of the stress distribution over the specimen cross section.

Thus, the fatigue limits in bending are considerably higher than those under axial loading, the local stresses at the stress concentrator tip that correspond to the fatigue limits are higher than the fatigue limits in the uniform stressed state.

The generalization of the investigation results is done either by constructing the empirical relationships that relate the fatigue limit value to the stress gradient [1], or by constructing the models that are based on certain hypotheses.

It is supposed in [2] that the difference in the fatigue limits in the uniform and nonuniform stressed states is governed by the difference between the nominal stresses calculated on the assumption of elastic deformation and the actual ones calculated taking into account the inelastic cyclic deformation in the nonuniform stressed state.

In the studies described in [3, 4, and others], the stress gradient effect is explained in terms of statistical strength theories.

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In the investigations of [5, 6, and others], the explanation of this effect is given in terms of critical distance theories, which assume that either stresses at some distance from the surface or averaged stresses in the surface layer are responsible for the fatigue fracture.

In [7–10, and others], the stress gradient effect is due to the difference in the inelastic deformation behavior of surface layers of the metal under conditions of uniform and nonuniform stressed states.

All the mechanisms that underlie the above hypotheses take place, to a certain extent, in real materials during fatigue fracture. With the use of the relationships that are based on one of these hypotheses, the consideration of the effect of other factors is achieved by the empirical selection of parameters in the chosen model.

Empirical relationships that relate the fatigue limit to the stress gradient are the most simple to use because there is no need to determine the parameters corresponding to different hypotheses, which is not an easy task.

At the same time, empirical relationships, many of which have been formulated quite a long time ago, need to be further improved taking into account new experimental data, with new approaches to generalizing experimental results being developed.

Starting from the generalization of the results taken from the literature and original experimental investigations performed under axial loading and in bending [11–30], this paper proposes and justifies the relationships for determining the fatigue limits of steels in the nonuniform stressed state, including those in the presence of the stress concentration, that are based on the use of the parameters of the mechanical properties of the material under study, relative stress gradient, theoretical stress concentration factor and fatigue characteristics in the uniform stressed state.

Investigation Results and Their Analysis. A characteristic of the nonuniform stressed state is either the stress gradient

$$\eta = \frac{d\sigma}{dx},$$

where σ is the stresses and x is the geometric size, or the relative stress gradient for the most stressed point that takes place on the specimen or stress concentrator surface under elastic deformation

$$\bar{\eta} = \frac{1}{\sigma_{\max}} \frac{d\sigma}{dx_{x=0}}.$$

In the case of elastic deformation in bending, the relative stress gradient is

$$\bar{\eta} = \frac{2}{h} \qquad \text{or} \qquad \bar{\eta} = \frac{2}{d},$$

where h is the height of the flat specimen and d is the diameter of the circular specimen.

In the stress concentrator, under axial loading, the relative stress gradient is given as

$$\overline{\eta}=\frac{2}{\rho},$$

and in bending it is given as

$$\overline{\eta}=\frac{2}{h}+\frac{2}{\rho} \qquad \text{or} \qquad \overline{\eta}=\frac{2}{d}+\frac{2}{\rho},$$

where ρ is the curvature radius in the concentrator.

In further analysis, the fatigue limits at 10^7 cycles with a symmetric load cycle for smooth specimens under axial loading σ_{-1} and in bending $(\sigma_{-1})_b$, as well as the fatigue limits (nominal stresses) of notched specimens under axial loading σ_{-1}^{nth} and in bending $(\sigma_{-1})_b^{nth}$ were considered.

The effective stress concentration factor was determined by the formulas:

$$K_\sigma=\frac{\sigma_{-1}}{\sigma_{-1}^{nth}} \qquad \text{or} \qquad K_\sigma=\frac{(\sigma_{-1})_b}{(\sigma_{-1})_b^{nth}}.$$

The ratio of the fatigue limits for smooth specimens in bending to those under axial loading is denoted by K'_σ.

In the analysis of the results of experimental investigations, as was shown in [31, 32, and others], account was taken of the fact that for the overwhelming majority of steels, cyclic inelastic strains at the stresses equal to the fatigue limit at 10^7 cycles to failure are not large, and the difference between the nominal and actual stresses is small, not exceeding 2–3%, which gives grounds to calculate stresses using formulas of elasticity theory. A similar picture is observed in the stress concentrator. Higher cyclic inelastic strains occur in high-ductility austenitic steels, which requires special consideration.

In the analysis of the relationship between fatigue limit and stress gradient, it is necessary to take into account the different nature of the stress distribution over the height of smooth specimens and those with stress concentrators. In concentrators, in areas of high stresses where the maximum stress gradients take place, the relative volume of the material is lower than in smooth specimens in bending, which leads to an increase in the local characteristics of the limit state.

Critical distance theories [5, 6, and others], which have been used in recent years to explain the effect of stress concentration on the fatigue strength, are based on the consideration of this factor.

In the process of the above generalization, we used 156 fatigue curves obtained experimentally and their corresponding fatigue limits for 40 grades of carbon and alloy steels at 10^7 cycles to failure in the nonuniform stressed state, including the fatigue limits in bending of smooth specimens with the relative stress gradients varying within $\overline{\eta}=0.2$ to 2.0; the fatigue limits in tension–compression of specimens with stress concentrators with the relative stress gradients and theoretical stress concentration factors varying within $\overline{\eta}=0.34$ to 15.5 and

$\alpha_\sigma = 1.56$ to 3.35, respectively, and, finally, the fatigue limits in bending of specimens with stress concentrators with the relative stress gradients and theoretical stress concentration factors varying within $\overline{\eta} = 1.352$ to 242.0 and $\alpha_\sigma = 1.25$ to 6.3, respectively. Experimental results for flat specimens with circular holes and side notches and cylindrical specimens provided with a circular recess were analyzed. All fatigue curves under analysis were obtained during the tests with a fully-reversed load cycle in the absence of manufacturing residual stresses in surface layers of both smooth and notched specimens. The diameter and thickness of specimens varied within 5 to 40 mm.

The fatigue limits found experimentally for some of the investigated steels in the nonuniform stressed state $(\sigma_{-1}^{nth})_e$ are presented selectively as an example in Table 1 using the data given in [13, 25, 26]. This same table gives the values of the theoretical and effective stress concentration factors $[\alpha_\sigma, (K_\sigma)_e]$, the relative stress gradient $(\overline{\eta})$, and also those of both the fatigue limits $(\sigma_{-1}^{nth})_c$ and the effective stress concentration factor $(K_\sigma)_c$ calculated using the method given below.

The ratio of the values of the fatigue limit to the effective stress concentration factor in tension is determined by relationship

$$\sigma_{-1}^{*} = \alpha_\sigma \sigma_{-1}^{nth} = f(\overline{\eta})\sigma_{-1}, \qquad \text{whence} \qquad K_\sigma = \frac{\sigma_{-1}}{\sigma_{-1}^{nth}} = \frac{\alpha_\sigma}{f(\overline{\eta})}, \tag{1}$$

and in bending is determined by relationship

$$\sigma_{-1}^{*} = \alpha_\sigma (\sigma_{-1})_b^{nth} = f(\overline{\eta})\sigma_{-1}, \qquad \text{whence} \qquad K_\sigma = \frac{\sigma_{-1}}{(\sigma_{-1})_b^{nth}} = \frac{\alpha_\sigma}{f(\overline{\eta})}, \tag{2}$$

where σ_{-1}^{*} is the fatigue limit (local stresses at the concentrator tip), $f(\overline{\eta})$ is the function defining the effect of the stress gradient, and α_σ is the theoretical stress concentration factor.

As a result of the generalization of the experimental data and the use of relations (1) and (2), the empirical relationships were obtained for the function $f(\overline{\eta})$, fatigue limits and effective stress concentration factors for the specimen geometry and modes of loading under consideration (Table 2).

The formulas given in Table 2 make it possible to determine the fatigue limits and effective stress concentration factors for specimens with stress concentrators in bending using not only the values of fatigue limits under axial loading, but also the bending test results.

Relationships (12) and (13) in Table 2 are derived by substituting the σ_{-1} values found from (6) and (7) into relations (10) and (11).

Numerical values of the parameters c_1 and c_2 were determined from the formulas for the fatigue limit given in Table 1 provided that the fatigue limits found experimentally and the calculated ones are equal.

Based on this analysis, it was assumed that for smooth specimens in bending $c_1 = 1.0$ for $\overline{\eta} < 1.0$ and $c_1 = 0.7$ for $\overline{\eta} \geq 1.0$.

Table 1

Calculation Results Based on Some Literature Data. Fatigue Limits of Steels in the Nonuniform Stressed State

Material, steel grade, $\sigma_{0.2}/\sigma_b$	Type of loading	$\bar{\eta}$, mm^{-1}	α_σ	$(\sigma_{-1})_e$, $(\sigma_{-1}^{nth})_e$, MPa	$(\sigma_{-1}^{nth})_c$, MPa	$(K_\sigma)_e$	$(K_\sigma)_c$	$(c_1)_e$, $(c_2)_e$	$(c_1)_c$, $(c_2)_c$
1	2	3	4	5	6	7	8	9	10
CSN 12010 [13] (normalization) $\sigma_{0.2}/\sigma_b = 0.634$	T–C	0	1.00	203.0	–	1.000	1.000	–	–
		0.340	2.18	110.1	105.0	1.844	1.933	0.680	0.466
		1.000	2.18	114.7	112.7	1.770	1.801	0.520	0.466
		1.000	1.56	173.1	157.6	1.173	1.288	0.770	0.466
		1.000	1.95	133.3	126.0	1.523	1.611	0.640	0.466
		1.000	2.26	115.0	108.8	1.765	1.866	0.640	0.466
		3.040	2.68	104.5	102.0	1.943	1.990	0.520	0.466
		7.300	2.75	112.7	111.0	1.801	1.829	0.490	0.466
		15.500	2.85	138.6	119.9	1.465	1.693	0.700	0.466
	B	2.000	1.00	290.0	286.4	1.430	1.410	0.740	0.700
		1.000	1.00	270.0	264.7	1.330	1.300	0.770	0.700
		0.500	1.00	270.0	265.2	1.330	1.310	1.100	1.000
		0.200	1.00	250.0	244.2	1.230	1.200	1.150	1.000
CSN 12010 [13] (heat-treated) $\sigma_{0.2}/\sigma_b = 0.718$	T–C	0	1.00	290.0	–	1.000	1.000	–	–
		0.340	2.18	160.6	167.4	1.806	1.732	0.800	1.000
		3.040	2.68	151.1	141.8	1.919	2.045	0.540	0.411
		7.800	2.75	160.0	154.5	1.813	1.877	0.460	0.411
	B	1.000	1.00	360.0	378.1	1.241	1.304	0.540	0.700
40Kh [26] $\sigma_{0.2}/\sigma_b = 0.407$	B	0.400	1.00	315.0	–	1.000	1.000	–	–
		5.400	2.05	180.0	189.1	1.750	1.666	0.535	0.634
		1.650	1.32	250.0	251.6	1.260	1.252	0.620	0.634
		0.933	1.20	285.0	288.0	1.105	1.094	0.960	1.000
		0.667	1.10	295.0	302.1	1.068	1.043	0.900	1.000
		0.100	1.00	275.0	–	1.000	1.000	–	–
		5.100	3.65	97.0	102.4	2.835	2.686	0.525	0.634
		1.350	2.05	160.0	154.1	1.719	1.785	0.765	0.634
		0.413	1.32	230.0	232.8	1.196	1.181	0.940	1.000
		0.050	1.00	255.0	–	1.000	1.000	–	–
		2.550	3.65	85.0	89.6	3.000	2.846	0.510	0.634
		1.300	2.55	113.0	118.7	2.257	2.148	0.500	0.634
		0.675	2.05	150.0	151.8	1.700	1.680	0.950	1.000
		0.206	1.32	210.0	210.6	1.214	1.211	0.980	1.000

Continued Table 1

1	2	3	4	5	6	7	8	9	10
45 [25] $\sigma_{0.2}/\sigma_b = 0.598$	B	0.266	1.000	309.0	–	1.000	1.000	–	–
		0.683	1.207	256.0	280.4	1.210	1.102	0.640	1.000
		1.330	1.444	214.0	212.3	1.480	1.455	0.510	0.491
		2.647	1.698	182.0	175.3	1.920	1.763	0.575	0.491
		3.599	1.931	160.0	159.3	2.190	1.940	0.500	0.491
		5.266	2.146	144.0	142.4	2.570	2.170	0.510	0.491
		7.673	2.414	128.0	127.7	3.020	2.420	0.495	0.491
		10.260	2.452	126.0	115.4	3.490	2.678	0.650	0.491
		0.133	1.000	286.0	–	1.000	1.000	–	–
		0.667	1.355	211.0	223.0	1.480	1.283	0.770	1.000
		1.331	1.713	167.0	159.6	1.920	1.792	0.620	0.491
		2.133	2.000	143.0	136.0	2.360	2.103	0.620	0.491
		2.633	2.270	126.0	127.7	2.570	2.240	0.460	0.491
		3.467	2.383	120.0	117.6	2.880	2.432	0.540	0.491
		5.133	2.698	106.0	104.7	3.400	2.732	0.52	0.491
		8.133	3.147	91.0	88.2	4.300	3.243	0.550	0.491
35 [26] $\sigma_{0.2}/\sigma_b = 0.47$	B	0.400	1.000	245.0	–	1.000	1.000	–	–
		1.650	1.320	185.0	192.2	1.324	1.275	0.485	0.585
		0.100	1.000	225.0	–	1.000	1.000	–	–
		20.100	6.300	60.0	59.3	3.750	3.794	0.610	0.585
		2.600	2.550	103.0	107.2	2.184	2.099	0.500	0.585
		1.350	2.050	120.0	124.0	1.875	1.815	0.500	0.585
		0.050	1.000	205.0	–	1.000	1.000	–	–
		1.300	2.550	90.0	93.8	2.278	2.186	0.470	0.585
		0.206	1.320	160.0	169.3	1.281	1.211	0.700	1.000
38Kh2N2MA [26] $\sigma_{0.2}/\sigma_b = 0.5$	B	0.400	1.000	320.0	–	1.000	1.000	–	–
		20.400	3.650	120.0	129.1	2.667	2.479	0.460	0.562
		10.400	2.550	155.0	164.8	2.065	1.942	0.460	0.562
		5.400	2.050	190.0	185.6	1.684	1.724	0.610	0.562
		0.100	1.000	290.0	–	1.000	1.000	–	–
		5.100	3.650	110.0	104.4	2.636	2.778	0.680	0.562
		2.600	2.550	140.0	136.9	2.071	2.118	0.620	0.562
		1.350	2.050	170.0	158.6	1.706	1.828	0.780	0.562
		0.050	1.000	265.0	–	1.000	1.000	–	–
		2.550	3.650	95.0	90.4	2.789	2.931	0.685	0.562
		1.300	2.550	130.0	120.4	2.038	2.201	0.800	0.562
		0.675	2.050	165.0	157.8	1.606	1.679	1.200	1.000

Note. “T–C” – tension–compression, “B” – bending; subscripts *e* and *c* correspond to the experimental and calculated values, respectively.

Table 2

Relationships for Determining the Fatigue Limits of Steels in the Nonuniform Stressed State

Test conditions	$f(\overline{\eta})$	Fatigue limit	Effective stress concentration factor
Bending of smooth specimens	$\sqrt{1+c_1\sqrt{\overline{\eta}}}$ (3)	$(\sigma_{-1})_b = \sigma_{-1}\sqrt{1+c_1\sqrt{\overline{\eta}}}$ (6)	$K'_\sigma = \dfrac{(\sigma_{-1})_b}{\sigma_{-1}} = \sqrt{1+c_1\sqrt{\overline{\eta}}}$ (7)
Tension–compression of specimens with a stress concentrator	$\sqrt{1+c_2\sqrt{\overline{\eta}}}$ (4)	$\sigma_{-1}^{nth} = \sigma_{-1}\dfrac{\sqrt{1+c_2\sqrt{\overline{\eta}}}}{\alpha_\sigma}$ (8)	$K_\sigma = \dfrac{\sigma_{-1}}{\sigma_{-1}^{nth}} = \dfrac{\alpha_\sigma}{\sqrt{1+c_2\sqrt{\overline{\eta}}}}$ (9)
Bending of specimens with a stress concentrator (calculation of the fatigue limit of smooth specimens in tension–compression)	$\sqrt{1+c_2\sqrt{\overline{\eta}}}$ (4)	$(\sigma_{-1})_b^{nth} = \sigma_{-1}\dfrac{\sqrt{1+c_2\sqrt{\overline{\eta}}}}{\alpha_\sigma}$ (10)	$K_\sigma = \dfrac{\sigma_{-1}}{(\sigma_{-1})_b^{nth}} = \dfrac{\alpha_\sigma}{\sqrt{1+c_2\sqrt{\overline{\eta}}}}$ (11)
Bending of specimens with a stress concentrator (calculation of the fatigue limit of smooth specimens in bending)	$\dfrac{\sqrt{1+c_2\sqrt{\overline{\eta}}}}{\sqrt{1+c_1\sqrt{\overline{\eta}}}}$ (5)	$(\sigma_{-1})_b^{nth} = (\sigma_{-1})_b\dfrac{\sqrt{1+c_2\sqrt{\overline{\eta}}}}{\alpha_\sigma\sqrt{1+c_1\sqrt{\overline{\eta}}}}$ (12)	$K_\sigma = \dfrac{(\sigma_{-1})_b}{(\sigma_{-1})_b^{nth}} = \dfrac{\alpha_\sigma\sqrt{1+c_1\sqrt{\overline{\eta}}}}{\sqrt{1+c_2\sqrt{\overline{\eta}}}}$ (13)

The value of c_2 is defined by relation

$$c_2 = 1 - \sigma_{0.2}/\sigma_b + 0.25(\sigma_{0.2}/\sigma_b)^2, \tag{14}$$

where $\sigma_{0.2}$ and σ_b are the 0.2 offset yield stress and the ultimate strength of the material under study, respectively.

Figure 1 presents the values of c_2 as a function of the ratio $\sigma_{0.2}/\sigma_b$ for the materials under consideration for different test types.

As follows from the above, the c_2-values which determine the dependence of the fatigue limit on the stress gradient for specimens with stress concentrators, are significantly lower than the c_1-values, which determine the dependence of the fatigue limit on the stress gradient for smooth specimens in bending.

The main difference of the relations in Table 2 from those given in the literature is that the function $f(\overline{\eta})$, which defines the dependence of the σ_{-1}^{*} values on the stress gradient, is not the same for smooth specimens and those with stress concentrators. This difference is defined by the difference in the parameters c_1 and c_2.

Figure 2 illustrates the dependences $f(\overline{\eta}) - \overline{\eta}$ shown as solid lines for smooth specimens in bending (*1*) and specimens with stress concentrators (*2*) that are calculated from the formulas in Table 2, together with the experimental data for different steels, for which the investigation results are presented as points.

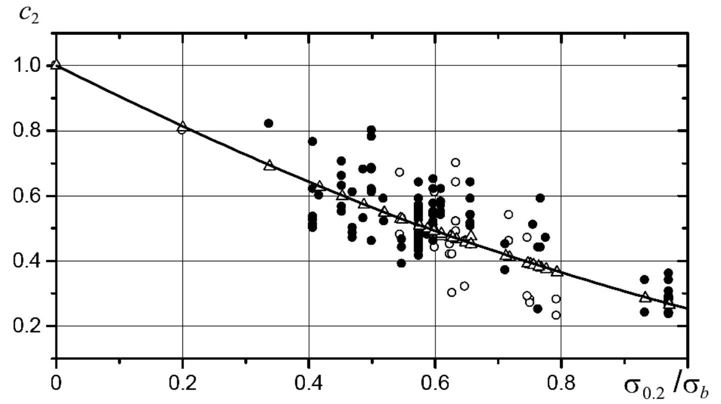
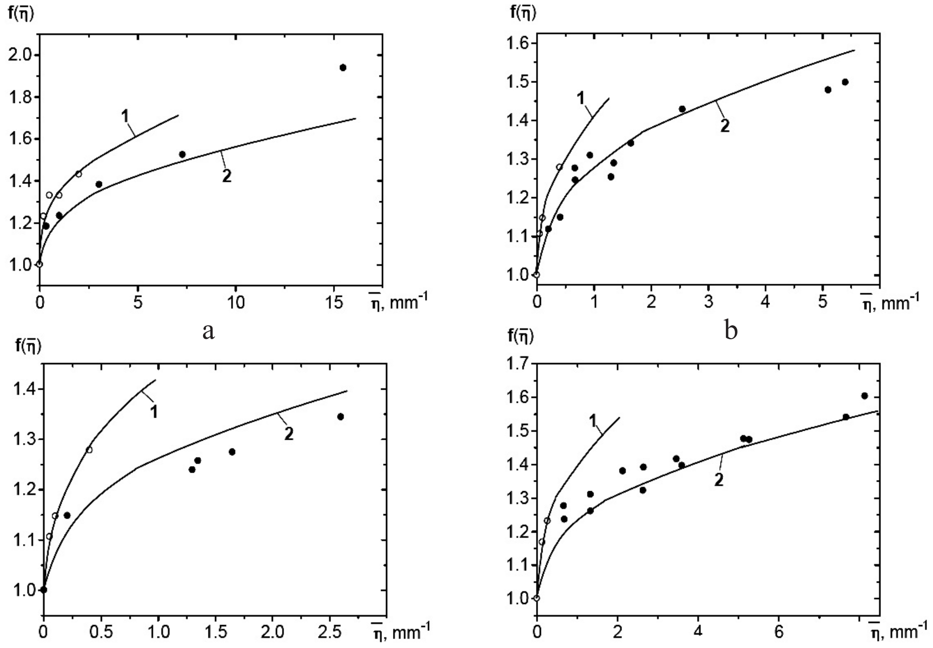


Fig. 1 Dependence of experimental values of the coefficient c_2 on the magnitude of the ratio $\sigma_{0.2}/\sigma_b$ for the materials under investigation: (*1*) experimental points for specimens with stress concentrators in tension–compression; (*2*) experimental points for specimens with stress concentrators in bending; (*3*) approximating curve according to relationship (14).



a

b

c

d

Fig. 2. Calculated dependences $f(\bar{\eta})-\bar{\eta}$ for smooth specimens in bending (*1*) and specimens with stress concentrators (*2*); experimental data (*I*, *II*) for different steels: (a) steel CSN 12 010 ($\sigma_{-1}=203$ MPa, $\sigma_{0.2}/\sigma_b=0.634$) [13]; (b) steel 40Kh [$(\sigma_{-1})_b=255$; 275; and 315 MPa, $\sigma_{0.2}/\sigma_b=0.407$] [26]; (c) steel 35 [$(\sigma_{-1})_b=205$; 225; and 245 MPa, $\sigma_{0.2}/\sigma_b=0.47$] [26]; (d) steel 45 [$(\sigma_{-1})_b=286$; 309 MPa, $\sigma_{0.2}/\sigma_b=0.598$] [25].

The presented results show that the calculated dependences $f(\bar{\eta})-\bar{\eta}$ for the same material (which is used in smooth specimens in bending and specimens with stress concentrators) differ appreciably. Experimental points obtained in the above-mentioned studies correspond to the calculated dependences.

It also follows from the plots given in Fig. 2 that the rate of increase in the function $f(\overline{\eta})$ with an increase in $\overline{\eta}$ is higher for smooth specimens than for specimens with stress concentrators, which is confirmed by the experimental data.

The analysis of the relationships presented in Table 2 shows that, according to them, the effective stress concentration factor increases with the ratio $\sigma_{0.2}/\sigma_b$.

Figure 3 presents the calculated dependences $K_\sigma - \sigma_{0.2}/\sigma_b$ for the stress concentrator in bending shown in this figure for various values of the theoretical stress concentration factor and their corresponding relative stress gradients. It follows from this figure that the K_σ-value increases with the ratio $\sigma_{0.2}/\sigma_b$.

This nature of the K_σ versus $\sigma_{0.2}/\sigma_b$ dependence was observed in many experimental investigations [33, 34].

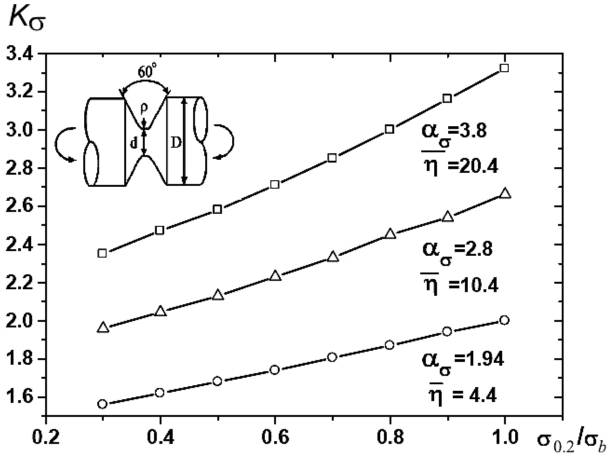


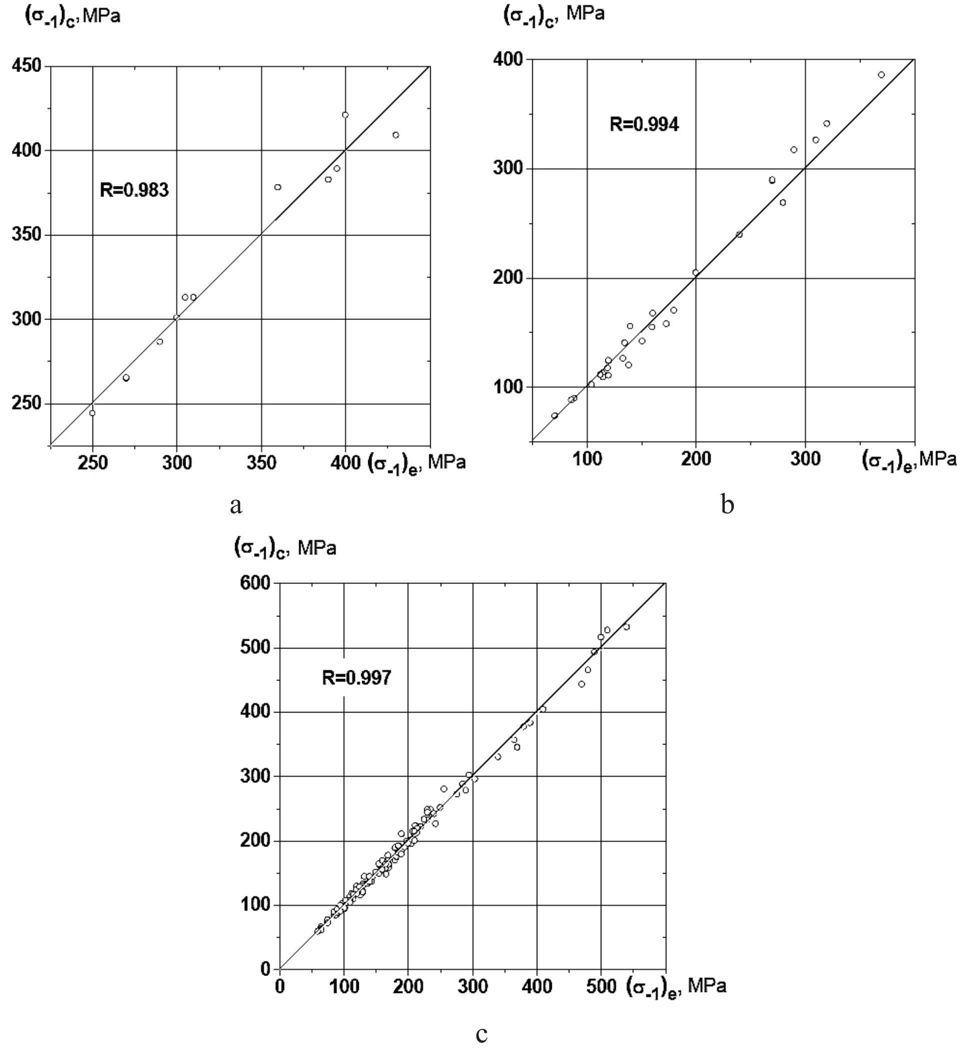
Fig. 3. Calculated dependences $K - \sigma_{0.2}/\sigma_b$ in bending for different values of α_σ and $\overline{\eta}$.

Figure 4 shows a comparison between the experimental and calculated values of the fatigue limits in the nonuniform stressed state determined by formulas (6), (8), (10), and (12). It is seen that the experimental and calculated values of the fatigue limits correspond to a single correlation relationship with a high correlation coefficient $R = 0.983$ to 0.997.

The fact that the above dependences describe equally well the results obtained in bending and tensile tests for flat specimens with stress concentrators when a linear stressed state takes place at their tips, and for cylindrical specimens with an annular groove in which a plane stressed state takes place at the concentrator tip can be explained by that, as was shown in [35], the equivalent stresses in these concentrators, according to the von Mises strength theory describing steel test results fairly well, do not much differ from the maximum normal stresses.

The results shown in Fig. 4 testify that the proposed relationships (6), (8), (10), and (12) are in good agreement with the experimental data and can be directly applied in the practice provided that the type of loading, values of the relative stress gradients and theoretical stress concentration factors, and certain mechanical properties of the material are known.

Conclusions. The experimental relationships between the fatigue limits of steels and effective stress concentration factors versus the stress gradient have been proposed and validated.



a b

c

Fig. 4. Comparison of the experimental and calculated values of the fatigue limits determined by formulas (6), (8), (10), and (12): (a) smooth specimens in bending; (b) specimens with stress concentrators in tension; (c) specimens with stress concentrators in bending.

In contrast to similar relationships available in the literature, the nature of the dependence of the fatigue limit on the stress gradient is assumed to be different for smooth specimens in bending and specimens with stress concentrators. A method for determining the parameters of the proposed relationships that takes into account the mechanical properties of steels under study has been validated. Good agreement has been shown between the proposed relationships and the experimental data.

Резюме

На основі узагальнення результатів численних експериментальних досліджень запропоновано і обґрунтовано емпіричні співвідношення між границями витривалості сталей і градієнтом напружень, у яких враховується накопи-

чення втомного пошкодження в гладких зразках та зразках із концентратором напружень. Запропоновано метод розрахунку параметрів цих співвідношень з урахуванням механічних властивостей сталей. Отримано хорошу збіжність між запропонованими співвідношеннями й експериментальними результатами.

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